Wind Shear Components

- This is to basically explain why the formula for components of the wind shear looks so strange.
- First I will quickly show that the obvious formula must be wrong.
- Then I will go into an explanation of how it should be.

Why obvious formula is incorrect?

Many people think that the formula for the x and y components of the wind shear should be:

$$\vec{\tau} = c_D \rho \begin{pmatrix} u_x^2 \\ u_y^2 \end{pmatrix}$$

where c_D is the drag coefficient, ρ is the air density and u_x and u_y are the wind components.

This formula must be wrong as the vector doesn't even point in the same direction as the wind! Try it with u_x=1 and u_y=2.

$$\vec{u} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

Furthermore, this formula is not rotationally invariant. The magnitude of this vector depends on the coordinate system you choose (assuming wind is indeed a vector)!!

Derive Correct Formula

What is wind shear? It is a measure of the force wind puts on the surface of the water. It is in the same direction as the wind and has magnitude:

$$\tau = c_D \rho u^2$$

A unit vector that points in the direction of the wind is:

$$\hat{u} = \frac{1}{u} \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

where u is:

$$u = \sqrt{u_x^2 + u_y^2}$$

Therefore, the formula for the wind shear vector is:

$$\vec{\tau} = \tau \hat{u} = c_D \rho u^2 \frac{1}{u} \begin{pmatrix} u_x \\ u_y \end{pmatrix} = c_D \rho u \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

This is the final, correct formula.

A Bit about the Drag Coefficient

There are a number of competing formulas for the drag coefficient. Measuring this coefficient is very difficult and depends somewhat on the method used. This is why there are several conventions.

The important thing, however, is to use one of them consistently. No one formulation seems to be that much better than any other, at least without a lot of investigation.